Comment on "Self-potential signals associated with preferential groundwater flow pathways in sinkholes" by A. Jardani, J. P. Dupont, and A. Revil

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Citation: Gibert, D., and P. Sailhac (2008), Comment on ''Self-potential signals associated with preferential groundwater flow pathways in sinkholes'' by A. Jardani, J. P. Dupont, and A. Revil, *J. Geophys. Res.*, *113*, B03210, doi:10.1029/2007JB004969.

1. Introduction

[1] *Jardani et al.* [2006a] (hereinafter referred to as JDR) present a self-potential survey showing circular anomalies associated with shallow sinkholes in a chalk karst. They perform various finite element modelings and data analysis, and, in particular, use a three-dimensional (3-D) version of the so-called dipole occurrence probability (DOI) tomography [*Iuliano et al.*, 2002a]. This method was proposed in its 2-D version by *Revil et al.* [2001] as a sequel of the charge occurrence probability (COP) tomography initially developed by *Patella* [1997a]. COP and DOI aim at producing an image of the probability density of the location of the causative sources located underground and causing the observed potential field anomalies. This probability is given as a function of depth and horizontal coordinates as shown in Figure 14 of JDR (and in Figure 1b). The paper by JDR is among the latest of a series where either COP or DOI is applied to electrical self-potential data [*Patella*, 1997a, 1997b; *Lapenna et al.*, 2003; *Revil et al.*, 2003; *Wilkinson et al.*, 2005; *Jardani et al.*, 2006b], electrical resistivity data [*Mauriello et al.*, 1998; *Mauriello and Patella*, 1999a; *Zhou and Greenhalgh*, 2002], electromagnetic prospecting [*Mauriello and Patella*, 1999b], and then gravity and magnetic data [*Mauriello and Patella*, 2001; *Iuliano et al.*, 2002a, 2002b; *Chianese and Lapenna*, 2007; *Saracco et al.*, 2007].

[2] In the present comment, we want to draw attention of the readers unaccustomed with the physics of potential fields that the appealing COP and DOI methods are based on some misconceptions. Hereafter, we explain why the affirmation made by JDR (paragraph 31) that "The DOI function represents therefore the probability of finding in a point of the subspace Ω a dipole responsible for the self-potential anomaly observed at the ground surface" is false. As a consequence, the claim by JDR (paragraph 30) to "provide algorithms to analyze quantitatively the self-potential signals in terms of source location and geometry of the source body" is doubtful.

[3] Before explaining and justifying our criticisms with detailed mathematical and physical considerations, we first point out several arguments which show that the DOI and COP concepts are questionable:

[4] 1. The DOI method consists in computing the so-called Source Occurrence Probability (SOP), η, recalled in equation (12) of JDR and in the equivalent equation (2) below. Unfortunately these equations reveal that the η function may take negative values and therefore does not satisfy the basic positiveness property necessary to give a chance to η to be a probability distribution. A response often made by users of SOP is that η is actually a signed probability whose sign is the one of the sources causing the analyzed potential field. Unfortunately, in what follows, we show that this argument is not sufficient to eliminate the negativity paradox and that η is actually not a probability density but instead must be seen as a normalized scalar product directly related to the upward continuation of potential fields.

[5] 2. The spurious probabilistic interpretation attributed to the η function may be further understood by considering the simple example of the SOP analysis of the anomaly created by an isolated dipole located at a point $\mathbf{p_s} = (x_s, y_s, z_s > 0)$ below the surface. If the potential field caused by this source is perfectly known in the horizontal plane $z = 0$ (as shown in Figure 1a), the recovery of the source location is an inverse problem which possesses a unique solution in $z > 0$. Obviously, for this particular case, the η function should restrict to either the 2-D or the 3-D Dirac distribution, $\delta(\mathbf{p} - \mathbf{p_s})$, in order to assign a probability of 1 at the source location $\mathbf{p_s}$ and 0 elsewhere. As can be seen in Figure 1b, the SOP is far from being equal to the expected Dirac distribution and instead spans a wide area. The failure of the SOP to correctly reproduce the location probability $\delta(\mathbf{p} - \mathbf{p_s})$ for this canonical problem questions the sense of the probabilistic role attributed to the SOP by several authors.

[6] 3. These doubts concerning the probabilistic role attributed to the SOP analysis are reinforced by the fact that no stochastic process is involved (e.g., Gaussian white noise, signal-to-noise ratio) in the mathematical derivations [e.g., *Patella*, 1997a] leading to the SOP method. This makes hard to consider the SOP as resulting from a

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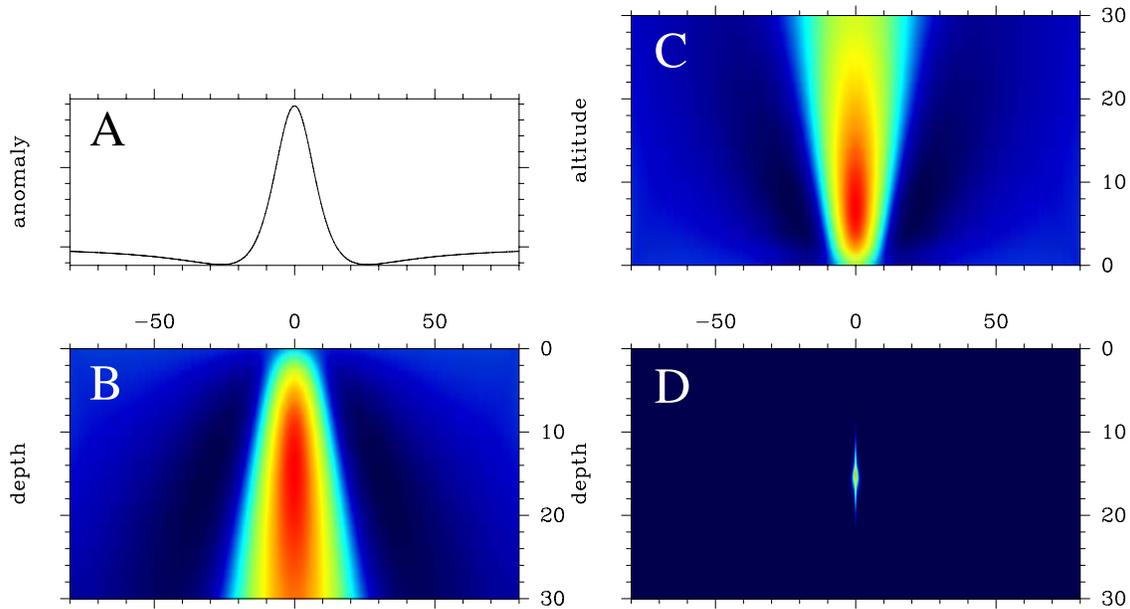
Figure 1. (a) Vertical component of the potential field caused by a vertical dipole located at $x_s = 0$ and $z_s = 15$ (arbitrary units). (b) DOI function corresponding to anomaly (Figure 1a). (c) Wavelet transform of the anomaly (Figure 1a) computed with an analyzing wavelet identical to the Green's function used to compute the DOI (Figure 1b). (d) Source function obtained by downward continuation of the wavelet transform (Figure 1c).

stochastic process by itself. Actually, the only argument raised by the advocates of SOP is that the η function is normalized (in absolute value). Let us recall that a normalized function cannot automatically be considered as a probability density function.

[7] We now turn on more technical and mathematical details in order to explain the real sense of the SOP function. We consider the 2-D case of spontaneous potential fields created in media with a homogeneous electrical conductivity. The simplifications have the advantage of resulting in the simplest formulas while preserving the fundamental physics of the problem. We use the framework of the continuous wavelet transform to show that the SOP is related to the wavelet transform of the analyzed potential field and that further mathematical treatment is necessary to obtain the source function [*Moreau et al.*, 1997, 1999; *Hornby et al.*, 1999]. Because of the minor differences between the DOI and COP methods (i.e., dipole versus monopole), both methods may be treated in the same theoretical framework.

2. Equivalence of SOP and Wavelet Transform

[8] In this section, we only recall the main formulas and mathematical steps necessary to show that the SOP function η is a particular version of the wavelet transform of the potential field [*Sailhac and Marquis*, 2001]. The reader is referred to our previous papers for further details and applications to various type of data and interpretation [*Moreau et al.*, 1997, 1999; *Sailhac et al.*, 2000; *Gibert and Pessel*, 2001; *Martelet et al.*, 2001; *Sailhac and Gibert*, 2003; *Boukerbout and Gibert*, 2006].

[9] As recalled by JDR, the SOP function is obtained by taking the scalar product, i.e., the cross correlation, between

the measured field, $\phi(x, y, z = 0)$ and a Green's function, $G(x, y|x_s, y_s, z_s)$, corresponding to both the measured physical quantity (e.g., spontaneous potential, gravity, etc.) and to the multipole nature of the point source chosen, i.e., a monopole for COP [*Patella*, 1997a] or a dipole for DOI [*Revil et al.*, 2001]. The results presented below are valuable in both two and three dimensions, but for clarity and conciseness of the discussion, we only write the 2-D expressions.

[10] The SOP is defined by (see *Patella* [1997a] for details)

$$\eta(x_s, z_s) = C \int_{-\infty}^{+\infty} \phi(x, z = 0) G(x|x_s, z_s) \mathrm{d}x, \qquad (1)$$

with $C = (E_\phi E_{G,zs})^{-1/2}$ a normalizing factor where E_ϕ and $E_{G,zs}$ are the energies of $\phi(x, z = 0)$ and $G(x|x_s, z_s)$, respectively.

[11] Considering the case of spontaneous potential treated by JDR and a constant electrical conductivity, the Green's function is invariant by translation and may be written as $G(x_s - x, z_s)$. Equation (1) then takes the form of a convolution product,

$$\eta(x_s, z_s) = C \int_{-\infty}^{+\infty} \phi(x, z = 0) G(x_s - x, z_s) \mathrm{d}x. \qquad (2)$$

Since the Green's function satisfies the Poisson equation, G is homogeneous. For instance, in case of dipole sources, G is homogeneous with degree -1 and reads

$$G(x, z) = \frac{1}{\pi} \frac{\alpha x + \beta z}{x^2 + z^2}, \qquad (3)$$

where $\mathbf{q} = (\alpha, \beta)$ is the direction of the dipole source.

[12] In the general 2-D case, a Green's function that satisfies the Poisson equation involves N successive derivations of the elementary Newtonian potential

$$G(x,z) = \mathcal{O}_{\mathbf{q_1}} \mathcal{O}_{\mathbf{q_2}} \cdots \mathcal{O}_{\mathbf{q_N}} \frac{1}{2\pi} \ln\left(x^2 + z^2\right), \qquad (4)$$

where $\mathcal{O}_{\mathbf{q}} = \mathbf{q} . \nabla$ is the derivative operator in direction $\mathbf{q}$, and the directions $\mathbf{q_i}$ depend on the multipolar nature of the causative source. This function is homogeneous with degree $\lambda = -N$ and thus verifies the following property:

$$\frac{1}{a} G(x/a, 1) = a^{\lambda-1} G(x, a), \quad \forall a > 0. \qquad (5)$$

[13] Using a dilation factor $a = z_s$ and inserting the homogeneity property (5) in the convolution formula (2) gives

$$\eta(x_s, z_s) = C z_s^{1-\lambda} \int_{-\infty}^{+\infty} \phi(x, z = 0) \frac{1}{z_s} G\left(\frac{x_s - x}{z_s}, 1\right) \mathrm{d}x, \qquad (6)$$

whose mathematical form is identical to that of a continuous wavelet transform [e.g., *Holschneider*, 1995]. This is illustrated in Figure 1c which represents the wavelet transform of the potential field shown in Figure 1a and where the Green's function G is taken as the analyzing wavelet.

3. Discussion

[14] We now stop the mathematical developments and turn to a discussion concerning the consequences of equation (6). The following considerations are borrowed from our papers cited above and where the mathematical proofs are detailed.

[15] A first important consequence directly related to the properties of the Green's function G is that the η function is a map of the upward continued and transformed potential field ϕ. We emphasize that this fact is rigorously mathematically established and not only a matter of difference of perception about the problem. The upward continuation offset is simply equal to z_s, and the transformation applied to ϕ depends on the choice of the Green's function (i.e., the source in the SOP terminology). For instance, the transformation corresponding to a horizontal dipole source is the horizontal derivative while it is a Hilbert transform for a vertical dipole source.

[16] Users of COP and DOI analysis assume (an indeed do not question) that the variable z_s corresponds to the source depth. Owing to the mathematical sense of η recalled in the preceding paragraph, this interpretation is confusing. Actually, when plotting the η function, the users of SOP are correct when they consider that z_s is the depth of the point source generating the Green's functions G. The mistake comes from the fact that the quantity, η, plotted at level z_s is neither a source term nor any downward continued quantity but simply the transformed potential field upward continued to the altitude z_s. In other words, the vertical axis must be directed upward instead of downward in the plots of the SOP function (compare Figures 1b and 1c).

[17] In the particular case shown in Figure 1a, where the analyzed potential field ϕ is the Green's function G itself, the SOP function of Figure 1b is (up to an unimportant normalization factor) the wavelet transform of a wavelet of the same family. In the wavelet terminology, this corresponds to the well-known reproducing kernel [e.g., *Holschneider*, 1995] whose shape is totally controlled by the mathematical nature of the analyzing wavelet considered (i.e., G) and not by the statistical distribution of the causative sources. Hence, giving a probabilistic sense to η is meaningless.

[18] Retrieving the source function involves a transformation of the wavelet transform, i.e., of the SOP function, and mainly involves a downward continuation which focuses the wavelet transform on the support of the sources in the negative-dilation domain, i.e., in the lower half plane corresponding to depth). This procedure is illustrated in Figures 1c and 1d where the source term is located at the apex of the conical pattern of the wavelet transform of the analyzed potential field shown above. Such a downward continuation is missing in SOP analysis. Because of the huge exponential action of the continuation operator [e.g., *Gibert and Galdéano*, 1985], the support of the source function is much more localized than the one of the η function. This shows differently that interpreting η as a probabilistic indicator for the locations of the causative sources as done by JDR, among others, is particularly hazardous.

[19] Users of the SOP method generally consider that when the Green's function is correctly chosen, the maximum of the η function provides the appropriate location of the sources. Indeed this is true for an isolated pole when using the Green's function for a pole, and for an isolated dipole when using the Green's function for a dipole. This is no longer valid when the homogeneity degrees of the source and of the Green's function differ, a situation often encountered with extended sources. For instance, in the case of a local pole, the depth of the maximum of the η function is the correct one when using a polar Green's function, but it is biased when using a dipole Green's function. These results are direct consequences of the properties of the reproducing kernels cited above. Indeed, this also explains why the η function (e.g., as cross correlation of local polar potential with polar Green's function) takes its anomaly of largest amplitude and widest shape in the vicinity of the source (assuming that the vertical axis remains oriented downward as SOP users do) and gives SOP users the impression that the SOP theory is correct. We invite them to keep in mind that the η function is a map of the upward continued potential field: as a classical consequence of the upward continuation operator, the anomaly is wide even when the source is local.

[20] As indicated above, the correct location of point source is obtained by applying a downward continuation to the wavelet transform, whatever the multipolar nature of the source and the homogeneity degree of the analyzing wavelet (i.e., G) [e.g., *Moreau et al.*, 1999]. The case of real extended source (i.e., other than point sources) is much more complicated because only the equivalent point source of an extended source is actually localized with the wavelet transform. In the general case, for complex (e.g., other than spherical) geometry of the source, the complex patterns of

the η function have to be interpreted through a Taylor or multipolar expansion of the wavelet transform [e.g., *Sailhac et al.*, 2000; *Sailhac and Gibert*, 2003; *Boukerbout et al.*, 2003]. Recovering the shape of an extended source from its localized equivalent multipolar sources is a nonunique inverse problem. For instance an infinitely number of concentric spheres of mass M may be assigned to a point mass M. Clearly, the η function is unable to correctly represent this nonuniqueness.

4. Conclusion

[21] We hope that this comment will help practitioners to understand the limits of the so-called COP and DOI methods used in many studies. In particular, we claim that (1) the probabilistic interpretation given to the η function has no justification and the probabilistic vocabulary should be avoided, (2) the η function is a particular wavelet transform of the analyzed SP data and, like the wavelet coefficients, η is not directly a tomography of the underground causative sources but an image in the upward continuation domain that shows some symmetries with underground sources, (3) as a consequence of the above points, the spatial extension of the η function has a complicated relationship with the real shape of the source function, which means that it cannot be plotted as if it was a tomography of the source like expected in the SOP method, and (4) it is our opinion that more traditional techniques of potential field analysis with more serious theoretical basis should be preferred to SOP methods. This is for instance the case of filtering techniques [e.g., *Boschetti et al.*, 2004; *Fedi et al.*, 2004].

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