Identification of sources of potential fields with the continuous wavelet transform: Application to VLF data

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[1] A method to localize and characterize the sources of VLF tilt anomalies is proposed. It relies on the continuous wavelet transform computed with particular analyzing wavelets possessing remarkable properties with respect to potential fields. An example with a synthetic dyke model shows how the method allows to locate the top of conductive structures both horizontally and vertically. An application to real data acquired over a conductive dyke illustrates the robustness of the method with respect to noise. *INDEX TERMS:* 0903 Exploration Geophysics: Computational methods, potential fields; 0925 Exploration Geophysics: Magnetic and electrical methods; 0619 Electromagnetics: Electromagnetic theory; 9820 General or Miscellaneous: Techniques applicable in three or more fields. **Citation:** Boukerbout, H., D. Gibert, and P. Sailhac, Identification of sources of potential fields with the continuous wavelet transform: Application to VLF data, *Geophys. Res. Lett.*, 30(8), 1427, doi:10.1029/2003GL016884, 2003.

1. Introduction

[2] The very-low frequency (VLF) method is a long-standing geophysical technique used to prospect the near-surface geology (see *Paal* [1965] for pioneering work and *McNeill and Labson* [1989] for a review). This technique works in the frequency domain and measures the magnetic field $\mathbf{H} = \mathbf{H}^p + \mathbf{H}^s$ above the ground, where $\mathbf{H}^s = (H_x^s, H_y^s, H_z^s)^t$ is the field produced by the electrically conductive Earth in response to the incident primary field $\mathbf{H}^p = (H_x^p, H_y^p, H_z^p)^t$. VLF waves are emitted by the navies transmitters which operate at frequencies 10 kHz < f <30 kHz. For distances larger than several wavelengths from the transmitter and in the flat-Earth approximation, the primary field is assumed independant of (x, y) and such that $\mathbf{H}^p = H_x^p\mathbf{x}$, where the $\mathbf{x}$ direction is perpendicular to the direction of propagation.

[3] The tilt $\Phi(f, x, y) = H_z^s/H_x$ is the main attribute used to detect conductivity variations with VLF data [e.g. *Fischer et al.*, 1983]. A first approximation is to assume $H_x^s \ll H_x^p$, so that

$$\Phi(f, x, y) \propto H_z^s(f, x, y). \tag{1}$$

A more precise analysis may be done by processing the tilt data to recover H_z^s [*Gharibi and Pedersen*, 1999]. Since the secondary field $\mathbf{H}^s$ is of internal origin, it may be derived in the frequency domain from a potential which verifies the Poisson equation with all sources located underground. Using equation 1 which shows that the vertical component of the secondary field is proportional to the VLF tilt data, techniques classically used to analyze potential field data like gravity and magnetics may directly be applied to VLF data.

[4] In previous studies, we presented a theoretical framework based on the continuous wavelet transform which allows an identification and characterization of the sources of potential fields [*Moreau et al.*, 1997, 1999; *Sailhac and Gibert*, 2003]. Other studies present application to aeromagnetic [*Sailhac et al.*, 2000], gravity [*Martelet et al.*, 2001], and self-potential data [*Gibert and Pessel*, 2001; *Sailhac and Marquis*, 2001]. In the present paper we show how the method can be applied to VLF data.

2. Wavelet Analysis of VLF Data

[5] In order to make the paper self-consistent, we briefly recall the main mathematical properties of the continuous wavelet transform $\mathcal{W}$ which is the mathematical basis of the method. A general presentation of the wavelet theory may be found in the book by *Holschneider* [1995] and a detailled discussion of the wavelet transform applied to potential field theory is given in *Moreau et al.* [1997, 1999], *Sailhac et al.* [2000], and *Sailhac and Gibert* [2003]. We define $\mathcal{W}$ as a convolution product,

$$\mathcal{W}[g, \phi_0](x, a) \equiv \int_{-\infty}^{+\infty} \frac{1}{a} g\left(\frac{x'}{a}\right) \phi_0(x - x') dx' = (\mathcal{D}_a g * \phi_0)(x), \tag{2}$$

where ϕ_0 is the function to be analyzed, g is the analyzing wavelet, and $\mathcal{D}_a$ is the dilation operator such that

$$\mathcal{D}_a g(x) \equiv \frac{1}{a} g\left(\frac{x}{a}\right), \tag{3}$$

where the dilation $a > 0$. In the general case, the analyzing wavelet g may be an oscillating complex function localized on the real line as shown in Figure 1. The oscillating behavior ensures that the wavelet has a vanishing integral and allows the analyzed signal ϕ_0 to be reconstructed from its wavelet transform. The localization, i.e. the fact that the wavelet has a

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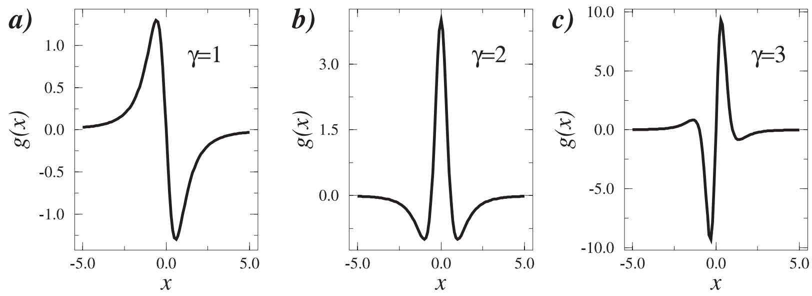


Figure 1. Examples of analyzing wavelets g. The γ parameter corresponds to the number of vanishing moments of the wavelet.

compact or quasi-compact support, enables the wavelet transform to perform a local analysis of the signal ϕ_0. From the point of view of signal processing, the dilated wavelets $\mathcal{D}_a g$ are band-pass filters with their bandwidth proportional to their central frequency which is inversely proportional to a. The wavelet transform is then constructed by stacking successive filtered versions of the original signal ϕ_0.

[6] In the present study, we use the mathematical properties of the wavelet transform with respect to homogeneous functions to localize the causative sources of H_z^s. An homogeneous function ϕ_0 of degree α is such that

$$\phi_0(\lambda x) = \lambda^\alpha \phi_0(x), \tag{4}$$

and its wavelet transform (2) reads:

$$\mathcal{W}[g, j\phi 0](\lambda b, \lambda a) = \lambda^\alpha \mathcal{W}[g, \phi_0](b, a). \tag{5}$$

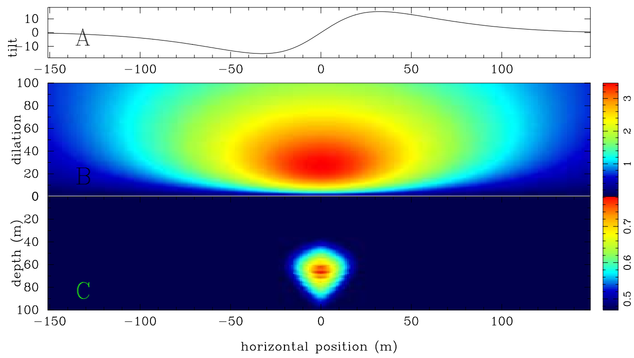
This equation shows that the entire wavelet transform of an homogeneous function may be extrapolated from $\mathcal{W}$ given at a single dilation. Geometrically, indicates that the wavelet transform of an homogeneous function has a cone-like appearence with its apex located on the a = 0 line at the homogeneity center of the analyzed function. The variation of the magnitude of the wavelet transform along any straight line crossing the homogeneity center is a power law of exponent α with respect to a [*Alexandrescu et al.*, 1995, 1996; *Holschneider*, 1995].

[7] In the present study, we shall assume that the analyzed potential field ϕ_0 (either Φ or H_z^s) is produced by an homogeneous source (monopole, dipole,...) localized at x_s and z_s. So, according to potential field theory, ϕ_0 is an upward-continued version of the homogeneous source. A particular class of wavelets exists such that the continuation property of the wavelet transform is equivalent to the analytical continuation of potential fields. For such wavelets, some examples of which are shown in Figure 1 the dilation a may equivalently be seen as an upward-continuation offset of the analyzed potential field [*Moreau et al.*, 1997, 1999; *Hornby et al.*, 1999; *Sailhac et al.*, 2000]. The cone-like structure of the wavelet transform is preserved with an apex now located at (x_s, $a = -z_s$) below the wavelet half-plane $a > 0$. Applying a proper scaling of the dilations a with respect to z_s, the power-law variation of the magnitude of the wavelet transform is restituted and the homogeneity degree α of the source can be determined.

3. Synthetic Examples

[8] We now illustrate the method with a synthetic example aimed at reproducing the geological situation of the case study presented in the next Section. The conductivity model corresponds to a conductive vertical dyke with a width of 20 m and a conductivity σ = 0.5 S.m^{-1} burried in a resistive half-space with σ = 0.001 S.m^{-1}. The depth to the top of the dyke is 60 m. The tilt Φ has been computed with a finite element method [*Wannamaker et al.*, 1985] along a profile centered on the dyke. The sampling interval is 5 m and the frequency f = 19600 Hz.

[9] The wavelet transform was computed with an analytical complex wavelet whose real part is shown in Figure 1a. The wavelet analysis was first applied to the tilt profile $\Phi(x)$ (Figure 2), both the modulus and the phase maps displays a cone-like pattern typical of the wavelet transform of a point-like homogeneous source. The locations of the



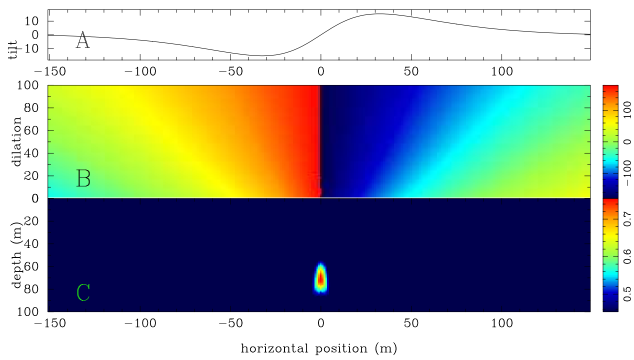


Figure 2. Synthetic dyke model. Top: (a) tilt $\Phi(x)$ computed for a conductive dyke located at x_s = 0 m and with a depth to top z_s = 60 m. (b) modulus of the wavelet transform of the tilt. (c) map of the ρ index showing a maximum at the location of the top of the dyke. Bottom: (b) phase of the wavelet transform of the tilt. (c) map of the ρ index.

apex of the cones are obtained by testing the cone-like geometry with respect to every point in a rectangular grid in the (x, z) lower half-plane. When a tested point corresponds to the apex of the cone, the modulus of the wavelet transform has a constant slope along any straight line emerging from the apex. This slope depends on the multipolar index α of the source. In a similar manner, the conical geometry of the phase of the wavelet transform may also be used to localize the source point. However, since the phase is constant along the cone lines, it cannot be used to derive the source multipolar index. The cone-like coherency is practically evaluated with an entropy criteria [*Tass et al.*, 1998],

$$\rho(x_s, z_s) \equiv \frac{\ln N + \sum_{i=1}^{N} h_i \ln h_i}{\ln N}, \tag{6}$$

where the h_i' s are the values of the normalized histogram of either the slopes of the modulus or the phases.

[10] The index ρ varies from 0 for a uniform histogram to 1 for a δ -like histogram. The map of the entropy index $\rho(x, z)$ computed from the modulus (top half of Figure 2) possesses a maximum ρ_{max} = 0.78 at (x_s, z_s) = (0.,65) m and $\alpha = -3.5$ indicating a quadrupolar source. The size of the ρ >0.6 area gives an idea of the uncertainty on the location of the source which, in the present instance, is about ±15 m. The phase map shown in the lower half of Figure 2 produces a sharper ρ peak and enables a better horizontal localization of the source at (x_s, z_s) = (0., 67) ms with ρ_{max} = 0.75. Both the modulus and the phase give very coherent localizations of the source which corresponds to the top part of the conductive dyke. This synthetic example also indicates that equation (1) is valid and that the tilt, which is the primary data provided by the VLF apparatus, may be directly analyzed.

4. Analysis of the Pont-Péan Data

[11] We now turn to field data acquired on the Pont-Péan (Britanny, France) fault-zone which is a mineralized diorite dyke which has been industrially exploited during the 18th and 19th centuries until a massive flood invaded the underground galleries in April 1904. The area has been extensively prospected and the geological structure is well-known, with a linear, north-south oriented, 4 km-long fault which enables a very good 2D approximation. Both the GBR and GBZ VLF antennas located in Great Britain were used since their northern location ensure good induction effects with a primary field $\mathbf{H}^p$ perpendicular to the stike of the fault. The VLF tilt Φ was measured every 5 meters along a profile oriented perpendicularly to the fault and where electrical tomography experiments and spontaneous potential measurements have already been done [*Gibert and Pessel*, 2001; *Pessel and Gibert*, 2003].

[12] The tilt profile obtained for f = 15975 Hz is shown in the top part of Figure 3 and displays a broad anomaly centered above the dyke whose thickness is known to be 30 m. The wavelet transform was computed for $1 \leq a \leq 100$, and it modulus map (top half of Figure 3) possesses a cone-like structure pointing toward a source point with ρ_{max} = 0.42 at (x_s, z_s) = (130, 46) m with $\alpha = -4.0$. The phase map (B in the lower half of Figure 3) also displays a clear cone

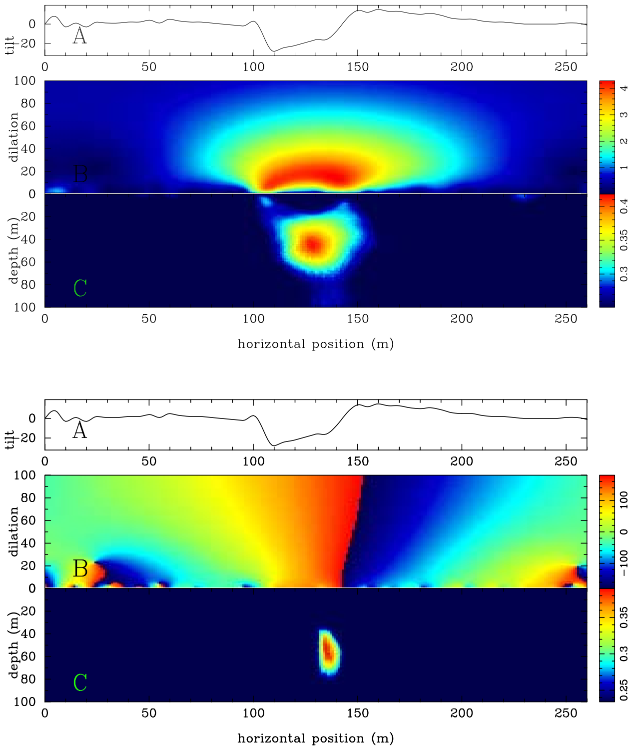


Figure 3. Wavelet zzanalysis of the Pont-Péan data. Top: (a) tilt profile measured perpendicularly to the dyke. (b) modulus of the wavelet transform of the tilt profile. (c) map of the ρ index derived from the modulus. Bottom: (b) phase of the wavelet transform. (c) map of the ρ index derived from the phase.

and its ρ map has a sharper maximum giving a source location (x_s, z_s) = (136, 57) m with ρ_{max} = 0.39.

[13] The results obtained from the analysis of both the modulus and the phase of the wavelet transform of the tilt Φ are self-consistent. The sharper localization of the source is obtained from the phase at (x_s, z_s) = (136 ± 5, 57 ± 20) m. This places the source just above the top of the prospected mineralized zone whose depth is about 80 m in this area. This result is in full agreement with previous electrical tomography experiments [*Pessel and Gibert*, 2003] which have shown that the mineralized dyke it overlained by a very conductive zone associated with water circulation and rock alteration and where induced currents may occur. So the VLF tilt anomaly is probably most produced by this shallower conductive source than by the deeper dyke.

5. Conclusion

[14] In this letter, we show how the continuous wavelet transform may be used to analyze VLF data. Application to other frequency-domain electromagnetic methods like magnetotelluric and loop-loop is straightforward. Application to time-domain methods is less trivial for the data must firstly be converted in the frequency domain. Provided the analyzing wavelet belongs to the semi-group of the Poisson equation, the continuous wavelet transform enables an easy localization of the sources responsible for the measured potential anomalies. The VLF tilt Φ may directly be

analyzed although back-transforming it into H_z^s may significantly reduce the noise level in some instances. The wavelet analysis gives an estimate of the location and the multipolar character of the source (dipole, etc.), and it must be kept in mind that these assumed homogeneous sources constitute the simplest model obtained from the potential data alone. This may constitute a prior information useful to initiate more sophisticated inversions with more physically realistic sources. This could, for instance, be derived by replacing the identified homogeneous sources with extended ones producing the same multipolar potential fields (e.g., a sphere is equivalent to a monopolar source).

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